

Adding and subtracting complex numbers

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In this unit we are going to look at how we can add and subtract complex numbers.

When you were at school you learnt how to add and subtract the counting numbers, that is the numbers 1, 2, 3, 4 and so on. Every time you met new sorts of numbers you learnt the process for adding and subtracting them. So, for example when you met fractions you learnt that to add two fractions you need to write them using a common denominator. Similarly there are rules for adding and subtracting complex numbers.

To add complex numbers we make use of a technique that you will have seen before when doing algebra. This involves collecting together **like terms**. We will start by adding two algebraic expressions.

Suppose we want to add

$$4 + 7t \quad \text{and} \quad 2 + 3t$$

The terms 4 and 2 are simply numbers. The terms $7t$ and $3t$ both contain t and are like terms. We collect the like terms together and simplify. So

$$\begin{aligned} 4 + 7t + 2 + 3t &= 4 + 2 + 7t + 3t \\ &= 6 + 10t \end{aligned}$$

Adding complex numbers works in exactly the same way.

Example

To add $4 + 7i$ and $2 + 3i$:

$$\begin{aligned} (4 + 7i) + (2 + 3i) &= 4 + 7i + 2 + 3i \\ &= 4 + 2 + 7i + 3i \\ &= 6 + 10i \end{aligned}$$

You will see that all we have done is added together the real parts and added together the imaginary parts of the two complex numbers to get the answer.

Example

To add $5 + 6i$ and $7 - 3i$:

$$\begin{aligned} (5 + 6i) + (7 - 3i) &= 5 + 6i + 7 - 3i \\ &= 5 + 7 + 6i - 3i \\ &= 12 + 3i \end{aligned}$$

Again note that all we have done is added together the real parts and added together the imaginary parts of the two complex numbers to get the answer.

Subtraction works in a very similar way:

Example

To subtract $2 + 3i$ from $4 + 7i$:

$$\begin{aligned}(4 + 7i) - (2 + 3i) &= 4 + 7i - 2 - 3i \\ &= 4 - 2 + 7i - 3i \\ &= 2 + 4i\end{aligned}$$

We have subtracted the real parts, and subtracted the imaginary parts.

Example

To subtract $7 - 3i$ from $5 + 6i$:

$$\begin{aligned}(5 + 6i) - (7 - 3i) &= 5 + 6i - 7 + 3i \\ &= 5 - 7 + 6i + 3i \\ &= -2 + 9i\end{aligned}$$

We have subtracted the real parts, and subtracted the imaginary parts.

In the next unit we will look at how to multiply two complex numbers together.